Robust efficiency methods on diviz and other news from PUT

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Data Envelopment Analysis - Running Example

\( D = \) a set of Decision Making Units

- \( i_1 \): number of doctors
- \( i_2 \): cost of doctor
- \( i_3 \): number of nurses
- \( i_4 \): cost of nurse

- \( o_1 \): outpatients
- \( o_2 \): inpatients

100 people per month
Essence of Data Envelopment Analysis

Inputs/outputs for DMU_0

\[
\begin{align*}
\text{i}_1 & \rightarrow x_{10} & \rightarrow y_{10} & \rightarrow o_1 \\
\text{i}_2 & \rightarrow x_{20} & \rightarrow \ldots & \rightarrow o_N \\
\text{i}_M & \rightarrow x_{M0} & \rightarrow \ldots & \rightarrow o_N \\
\end{align*}
\]

Efficiency measure

\[
E_0(v,u) = \frac{\sum_{n=1}^{N} u_n y_{n0}}{\sum_{m=1}^{M} v_m x_{m0}}
\]

<table>
<thead>
<tr>
<th>Inputs/outputs for DMU_0</th>
<th>Efficiency measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>\text{i}_1: number of doctors</td>
<td>22</td>
</tr>
<tr>
<td>\text{i}_2: cost of doctor</td>
<td>8</td>
</tr>
<tr>
<td>\text{i}_3: number of nurses</td>
<td>151</td>
</tr>
<tr>
<td>\text{i}_4: cost of nurse</td>
<td>100</td>
</tr>
<tr>
<td>o_1: outpatients</td>
<td>110</td>
</tr>
<tr>
<td>o_2 inpatients</td>
<td>90</td>
</tr>
</tbody>
</table>

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Lisbon, September 26, 2018
Value-based Efficiency Analysis

- Dias et al., *Journal Oper Res Soc*, 2006-2013
- Preference information: linear weight constraints
- Preference model: additive value functions

DMU₀ is efficient iff it attains the greatest comprehensive value for some value function.

\[ U(a) = \sum_{j=1}^{m} w_ju_j(a) \]

**Weight constraints:**
- \( w_{i1} \geq 2w_{i3} \)
- \( w_{i4} \geq 2w_{i2} \)
- \( w_{o1} \geq 2w_{o2} \)

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DMU0 is efficient iff it attains the greatest comprehensive value for some value function

\[ U(a) = \sum_{j=1}^{m} w_j u_j(a) \]

- Dias et al., *Journal Oper Res Soc*, 2006-2013
- DMU0 is efficient iff it attains the greatest comprehensive value for some value function
- analysis of **most favourable weights** (not unique)
- extremely **small share of weights** is analyzed (others neglected while being equally desirable)
- results **sensitive** to removal or inclusion of a single DMU
- indication of efficient/inefficient units (DEA does **not discriminate** among them)
- no imprecision in the specification of inputs/outputs and models
Value-based Efficiency Analysis - Extensions

- Apply all feasible input/output weights (value functions)
- No assumptions with respect to the production possibilities beyond the set of DMUs under consideration
- Results derived from pairwise comparisons are less sensitive to outliers

Three perspectives for robustness analysis:
- Efficiency scores
- Efficiency ranks
- Pairwise efficiency preference relation

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Number of doctors: $u_{i1}(a)$

Outpatients: $u_{o1}(a)$

Space of allowed marginal value functions

Iprecise inputs/outputs

H1: 8-10
H2: 2-3
H12: 4-5

Number of doctors:
- $i_1(a) = 10, 25, 40, 70$
- $u_{i1}(a) = 1, 0.5, 0$

Outpatients:
- $o_1(a) = 70, 150, 200, 250, 320$
- $u_{o1}(a) = 0, 0.3, 0.8, 1$

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Robustness analysis with Linear Programming

- Determine in **an exact way** results confirmed for all, some, the most and the least advantageous scenario
- Often not conclusive enough (what is certain is very rare, the space between the extreme cases if often large)

Robustness analysis with Monte Carlo simulation

- Apply Monte Carlo simulation (e.g., Hit-And-Run) **to provide stochastic indices**
- How probable are particular results / what is their distribution
- Stochastic indices estimated through simulation are not exact
Robustness Analysis - Efficiency scores

- Maximal efficiency for each DMU₀: \( E₀^* \)
- Minimal efficiency for each DMU₀: \( E₀^* \)
- For each DMU₀, we get an efficiency interval \([E₀^*, E₀^*]\)
- Maximal and minimal difference with respect to efficient DMU

Efficiency distribution:

<table>
<thead>
<tr>
<th>Efficiency acceptability interval index ( E_{AII}(DMU₀,bᵢ) )</th>
<th>Efficiency buckets ( bᵢ )</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0.0,0.1]</td>
<td>(0.1,0.2]</td>
</tr>
<tr>
<td>(0.2,0.3]</td>
<td>...</td>
</tr>
<tr>
<td>(0.9,1.0]</td>
<td></td>
</tr>
</tbody>
</table>

\( E_{AII}(DMU₀,bᵢ) = \) share of feasible scenarios for which \( DMU₀ \) attains efficiency in the interval \( bᵢ \)

- Minimal and maximal observed efficiency
- Estimate of an expected efficiency
Robustness Analysis - Pairwise Efficiency Relation

- **Necessary** relation \( \succeq \frac{N}{E} \) for a pair (DMU\(_0\), DMU\(_k\)) holds if \( U_0(i,o) \geq U_k(i,o) \) for all \((U,i,o) \in (S_U,S_{i,o})\).
- **Possible** relation \( \succeq \frac{P}{E} \) for a pair (DMU\(_0\), DMU\(_k\)) holds if \( U_0(i,o) \geq U_k(i,o) \) for all \((I,i,o) \in (S_U,S_{i,o})\).

- Pairwise efficiency outranking index \( \text{PEOI}(DMU_0,DMU_k) \) for each pair (DMU\(_0\),DMU\(_k\))
  \[
  \text{PEOI}(DMU_0,DMU_k) = \text{share of feasible scenarios for which DMU}_0 \text{ attains efficiency not worse than DMU}_k
  \]
- Pairwise efficiency winning index \( \text{PEWI}(DMU_0,DMU_k) \)
Robustness Analysis - Efficiency Ranks

- The best rank for DMU\(_0\): \(R^*_0\)
- The worst rank for DMU\(_0\): \(R_{0,*}\)
- For each DMU\(_0\), we get efficiency interval \([R^*_0, R_{0,*}]\)
- Efficiency rank acceptability index \(\text{ERAI}(\text{DMU}_0)\) for each DMU\(_0\)
  \(\text{ERAI}(\text{DMU}_0,k) = \) share of feasible scenarios for which DMU\(_0\) attains rank \(k\)
- For each DMU\(_0\): \(\sum_k \text{ERAI}(\text{DMU}_0,k) = 1\)
- Estimate of an expected rank
😊 TIME FOR DEMO
Robustness analysis with ratio-based efficiency model
- The most popular efficiency model used in DEA
- Imprecise models (constraints on weights)
- Imprecise inputs/outputs
- Three perspectives for robustness analysis
- Exact and stochastic analysis

Hierarchy of input/outputs
- Everything is ready
- Action required from D2: hierarchy in XMCDA

Airport efficiency
- Passengers
- Cargo

Data Envelopment Analysis on diviz
Integrated framework for robustness analysis using ratio-based efficiency model with application to evaluation of Polish airports

Miłosz Kadziński, Anna Labijak, Małgorzata Napieraj

Download links

- DEAPolishAirports.dvz for results without considering weight constraints;
- DEAPolishAirportsWithConstraints.dvz for results when considering weight constraints;
- DEAPolishAirportsWithoutOutlier.dvz for results when considering the set of airports without outlier (WAW).

Click here for a detailed description on how to import the workflow into diviz.
• Target non-European researchers: DEMATEL, VIKOR, BWM, etc.
• Advance MCDA R package with basic ELECTRE and PROMETHEE
• Comparative measures for ranking and sorting problems
• Clean up ordinal regression methods
• Graphical modules (rankings’ comparison, choice problem (kernel), etc.)