EWG MCDA
MULTIPLE CRITERIA
DECISION AIDING

POLITECHNIKA POZNAŃSKA
POZNAN UNIVERSITY OF TECHNOLOGY

Robust efficiency methods on diviz

and other news from PUT

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Data Envelopment Analysis - Running Example

***D* = a set of Decision Making Units**

inputs

- **i_1: number of doctors**
- **i_2 : cost of doctor**
- **i_3: number of nurses**
- **i_4: cost of nurse**

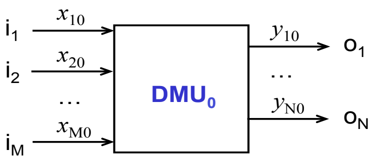
outputs

- **o_1: outpatients**
- **o_2 : inpatients**

100 people per month

Essence of Data Envelopment Analysis

Inputs/outputs for DMU_0

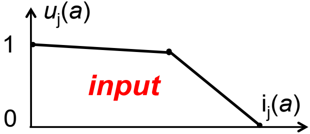


Efficiency measure

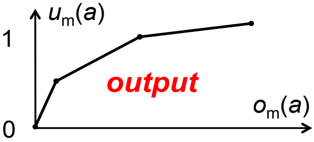
$$E_0(v,u) = \frac{\sum_{n=1}^{N} u_n y_{n0}}{\sum_{m=1}^{M} v_m x_{m0}}$$

	H1	H2		H12	WEIGHTS
i_1: number of doctors	22	19		38	v_1
i_2 : cost of doctor	8	2	.	4	v_2
i_3: number of nurses	151	131	.	284	v_3
i_4: cost of nurse	100	80	.	70	v_4
o_1: outpatients	110	150		250	u_1
o_2 inpatients	90	50		120	u_2

Value-based Efficiency Analysis

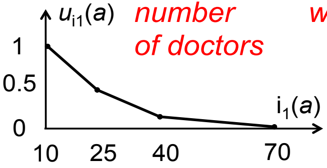


$$U(a) = \sum_{j=1}^{m} w_j u_j(a)$$



- Dias et al., *Journal Oper Res Soc*, 2006-2013
- Preference information: linear weight constraints
- Preference model: **additive value functions**

  DMU_0 is efficient iff it attains the greatest comprehensive value for some value function

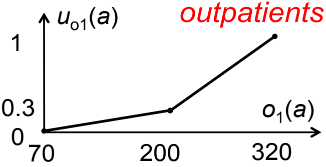


weight constraints

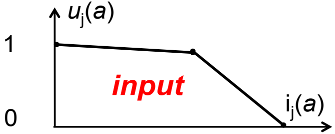
$w_{i1} \geq 2w_{i3}$

$w_{i4} \geq 2w_{i2}$

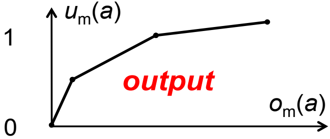
$w_{o1} \geq 2w_{o2}$



Value-based Efficiency Analysis - Criticism



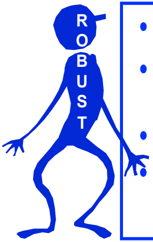
$$U(a) = \sum_{j=1}^{m} w_j u_j(a)$$



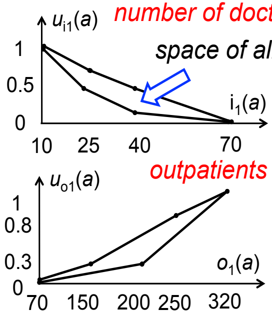
- Dias et al., *Journal Oper Res Soc*, 2006-2013
- DMU_0 is efficient iff it attains the greatest comprehensive value for some value function

- analysis of **most favourable weights** (not unique)
- extremely **small share of weights** is analyzed (others neglected while being equally desirable)
- results **sensitive** to removal or inclusion of a single DMU
- indication of efficient/inefficent units (DEA does **not discriminate** among them)
- no imprecision in the specification of inputs/outputs and models

Value-based Efficiency Analysis - Extensions



- Apply all feasible input/output weights (value functions)
- No assumptions with respect to the production possiblities beyond the set of DMUs under consideration
- Results derived from pairwise comparisons are less sensitive to outliers
- Three perspectives for robustness analysis:
  - Efficiency scores
  - Pairwise efficiency preference relation
  - Efficiency ranks

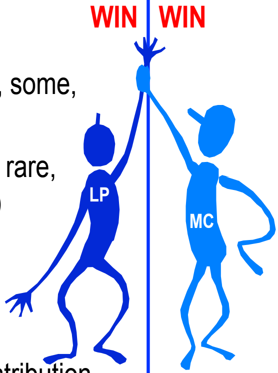


space of allowed marginal value functions

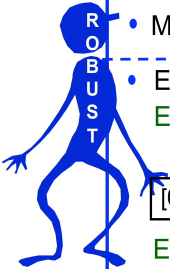
	H1	H2	...	H12
i_2	8-10	2-3	...	4-5
o_1	110-120	150-165	...	250-260

imprecise inputs/outputs

- **Robustness analysis with Linear Programming**
  - Determine in **an exact way** results confirmed for all, some, the most and the least advantageous scenario
  - Often not conclusive enough (what is certain is very rare, the space between the extreme cases if often large)
- **Robustness analysis with Monte Carlo simulation**
  - Apply Monte Carlo simulation (e.g., Hit-And-Run) **to provide stochastic indices**
  - How probable are particular results / what is their distribution
  - Stochastic indices estimated through simulation are not exact



Robustness Analysis - Efficiency scores



- Maximal efficiency for each DMU_0: E_0^*
- Minimal efficiency for each DMU0: $E_{0,*}$

LP

- For each DMU_0, we get an efficiency interval $[E_{0*}\ E_0^*]$
- Maximal and minimal difference with respect to efficient DMU

- Efficiency distribution:

  Efficiency acceptability interval index **EAII(DMU_0,b_i)**

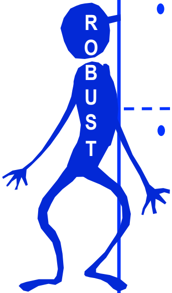
MC

efficiency buckets b_i

[0.0,0.1]	(0.1,0.2]	(0.2,0.3]	…	(0.9,1.0]

EAII(DMU_0,b_i) = share of feasible scenarios for which DMU_0 attains efficicicency in the interval b_i

- Minimal and maximal observed efficiency
- Estimate of an expected efficiency



- Necessary relation $\succeq_E^N$ for a pair (DMU_0, DMU_k) holds if $U_0(i,o) \geq U_k(i,o)$ for all (U,i,o) $\in$ (S_U,$S_{i,o}$)

LP 

- Possible relation $\succeq_E^P$ for a pair (DMU_0, DMU_k) holds if $U_0(i,o) \geq U_k(i,o)$ for all (I,i,o) $\in$ (S_U,$S_{i,o}$)

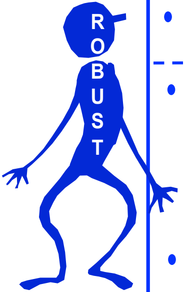
- Pairwse efficiency outranking index PEOI(DMU_0,DMU_k) for each pair (DMU_0,DMU_k)

MC 

PEOI(DMU_0,DMU_k) = share of feasible scenarios for which DMU_0 attains efficiency not worse than DMU_k

- Pairwse efficiency winning index PEWI(DMU_0,DMU_k)

Robustness Analysis - Efficiency Ranks



- The best rank for DMU_0: R_0^*

- The worst rank for DMU_0: $R_{0,*}$
- For each DMU_0, we get efficiency interval $[R_0^*, R_{0,*}]$

- Efficiency rank acceptability index $ERAI(DMU_0)$ for each DMU_0

  $ERAI(DMU_0,k)$ = share of feasible scenarios for which DMU_0 attains rank k

- For each DMU_0: $\sum_k ERAI(DMU_0,k) = 1$
- Estimate of an expected rank

☺ **TIME FOR DEMO**

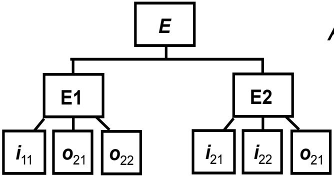
Robustness analysis with ratio-based efficiency model

- The most popular efficiency model used in DEA
- Imprecise models (constraints on weights)
- Imprecise inputs/outputs
- Three perspectives for robustness analysis
- Exact and stochastic analysis

$$E_0(v,u) = \frac{\sum_{n=1}^{N} u_n y_{n0}}{\sum_{m=1}^{M} v_m x_{m0}}$$

Hierarchy of input/outputs

- Everything is ready
- Action required from D2: hierarchy in XMCDA



Airport efficiency

- *Passengers*
- *Cargo*

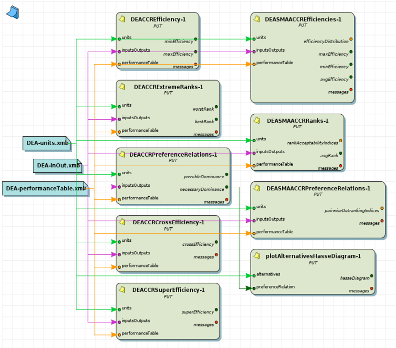
ELSEVIER

Omega

Volume 67, March 2017, Pages 1-18

Integrated framework for robustness analysis using ratio-based efficiency model with application to evaluation of Polish airports ☆

Miłosz Kadziński, Anna Labijak, Małgorzata Napieraj



Download links

- DEAPolishAirports.dvz for results without considering weight constraints;
- DEAPolishAirportsWithConstraints.dvz for results when considering weight constraints;
- DEAPolishAirportsWithoutOutlier.dvz for results when considering the set of airports without outlier (WAW).

Click *here* for a detailed description on how to import the workflow into diviz.

PUT's Next Steps related to Decision Deck

- Target non-European researchers: DEMATEL, VIKOR, BWM, etc.
- Advance MCDA R package with basic ELECTRE and PROMETHEE
- Comparative measures for ranking and sorting problems
- Clean up ordinal regression methods
- Graphical modules (rankings' comparison, choice problem (kernel), etc.)